What is an MTF? ... and why you should care?*

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Introduction
Modulation Transfer Function (MTF) is the scientific means of evaluating the fundamental spatial resolution performance of an imaging system, or components of that system. The purpose of this article is to introduce the MTF concept and its use, and to show why it is a superior way of specifying spatial resolution compared to other methods such as dots-per-inch (dpi) or visual bar target readings.

What is resolution and how does it relate to MTF?
Resolution is a measure of how well spatial details are preserved. The measuring of two items are required to define it, (1) spatial detail and (2) preservation. It is these fundamental metrics of detail and preservation that define MTF.

These detail and preservation metrics are not single measurements, but rather a continuum of measurements, which is why a functional curve quantifying them, i.e., the MTF, can be plotted. A range of spatial details is characterized by how well each and every one of those details is preserved, some better than others. An entire set of point pairs are then plotted; "spatial detail" on the x-axis and "extent of preservation" of that detail on the y-axis, to form the MTF as illustrated in Fig.1.

![Figure 1](image-url)

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One of the beauties of MTF is that it provides a continuum of unique rankings by which to judge a device's resolution performance. For the indecisive this may pose a problem, but for the analyst, this is gold.

**How is MTF measured?**

As stated above, two items are required for defining the MTF, (1) a measure of spatial detail, called frequency, and (2) a fundamental measure for determining how that detail is preserved, called modulation transfer. These will be addressed in that order.

**Frequency** - Spatial detail can be measured by the spatial frequency content of a given feature. A good example of this is the frequency of the line features in any 5 line bar target group. Each of the labeled line groups in a target indicates the spatial frequency for that group. For instance, the group labeled 2.0 has a frequency of 2.0 line pairs per millimeter (2.0 lp/mm). A line pair (dark-line/white-space combination) is more universally referred to as one cycle. For this example then, one cycle (line pair) spans $1/2.0 = 0.50$ mm. The higher the frequency, the greater the detail, the greater the number of cycles per unit distance, and the more closely spaced the lines become.

In scientific lexicon, the lines in the bar target cited above are called square-wave signals. This term derives from the square corners of their light intensity profile, as shown in the left hand side of Fig. 2 below. Though square wave signals are easy to manufacture into targets and can be used to measure MTF when properly treated, alone, they are not considered to be basic image building blocks. As such it is technically incorrect to use them as reference signals in determining MTFs. Instead, sine-waves are used. An example of a sine-wave signal compared with a square-wave of the same frequency is also shown in Fig. 2. Below each image is a cross-section plot of their light intensity profiles.

![Square-waves vs Sine-waves](image-url)

**Figure 2**
Sine-wave frequencies, usually in units of cycles/mm, are used as the metric for specifying detail in an MTF plot. These frequencies are always plotted as the independent variable on the x-axis. To complete the MTF metric, a measure of how well each sine-wave frequency is preserved after being imaged, i.e., transferred through an imaging device, is required. This measure, called modulation transfer, is plotted along the y-axis for each available frequency and completes the specification of MTF.

Modulation Transfer - The modulation for any signal is defined with two variables of that signal, the maximum light intensity value, $I_{\text{max}}$, and minimum light intensity value, $I_{\text{min}}$. Modulation is formulated as the quotient of their differences to their sums, as follows

$$\text{Modulation} = \frac{(I_{\text{max}} - I_{\text{min}})}{(I_{\text{max}} + I_{\text{min}})}$$

Figure 3 illustrates its calculation for a sine wave signal of 70% modulation. Note that its calculation is done in light reflectance, not density. Reflectance is considered a proper linear space in which to make the calculation.

The goal in determining MTF is to measure how well the input modulation is preserved after being imaged or in some way acted upon. This modulation transfer is quantified by comparing the modulations of the output sine waves after being imaged to the target's input sine-wave modulations before being imaged. This comparison is a simple ratio of output modulation to input modulation. It is formulated as

$$\text{Modulation Transfer} = \frac{(\text{Output Modulation})}{(\text{Input Modulation})} = \frac{(M_o)}{(M_i)}$$

The modulation transfer calculation is now complete. For a number of known frequencies, f, with known input modulations, $M_i$, an output modulation, $M_o$, can be determined after imaging. The ratio of $M_o/M_i$ is then plotted for each frequency. The resulting curve is the Modulation Transfer Function, or MTF. It is also sometimes called the Signal Frequency Response, or SFR.
An Example
Suppose one wished to determine the MTF of a desktop document scanner. A straightforward way of doing this is through the use of a sine-wave target. A good one will have a range of sine-wave frequencies along with solid gray patch areas used for calibration. The target will have supporting literature regarding the frequencies of each sine wave, their modulation, and the reflectances of the gray patches.

One would place the target on the platen, ensure it is properly aligned, choose a scanning resolution, and perform an 8-bit/channel (256 gray levels) scan. One now has a digital image of that target with potential count values ranging from 0 through 255 for each color channel. The calibration patches would then be used to ensure that the data was linear with respect to count value. For simplicity, the green channel alone will be analyzed. With a digital image analysis program such as Adobe Photoshop®, one would determine the average minimum and average maximum count value associated with the darkest and lightest areas of each sine wave frequency. These minimum and maximum count values are used to determine the output modulation, $M_o$, for each of the available frequencies. The modulation transfer would then be calculated. In tabular form these calculations appear as follows.

<table>
<thead>
<tr>
<th>Frequency (cycles/mm)</th>
<th>Target Modulation ($M_i$)</th>
<th>Avg Max. count value (out)</th>
<th>Avg. Min. count value (out)</th>
<th>Output Modulation ($M_o$)</th>
<th>Modulation Transfer ($M_o/M_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.70</td>
<td>254</td>
<td>46</td>
<td>0.69</td>
<td>0.98</td>
</tr>
<tr>
<td>3.0</td>
<td>0.70</td>
<td>234</td>
<td>66</td>
<td>0.56</td>
<td>0.85</td>
</tr>
<tr>
<td>5.0</td>
<td>0.65</td>
<td>201</td>
<td>99</td>
<td>0.34</td>
<td>0.53</td>
</tr>
<tr>
<td>6.0</td>
<td>0.63</td>
<td>187</td>
<td>112</td>
<td>0.25</td>
<td>0.40</td>
</tr>
<tr>
<td>7.0</td>
<td>0.58</td>
<td>170</td>
<td>130</td>
<td>0.13</td>
<td>0.22</td>
</tr>
<tr>
<td>9.0</td>
<td>0.55</td>
<td>154</td>
<td>145</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>10.0</td>
<td>0.50</td>
<td>153</td>
<td>147</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>11.0</td>
<td>0.46</td>
<td>151</td>
<td>148</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>12.0</td>
<td>0.44</td>
<td>150</td>
<td>150</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

A plot of Modulation Transfer vs Frequency yields the MTF for the scanner with the selected settings (i.e., dpi, sharpening, bit depth #, etc.) as illustrated in Fig. 4 below.
**How is the MTF curve interpreted?**

The MTF shape of Fig. 4 is typical for a well-behaved optical imaging device such as a document scanner. It begins with near unity modulation at zero-frequency and gradually becomes lower towards higher frequencies, until at some point the modulation becomes zero and remains so for all higher frequencies.

A simple way to interpret the modulation transfer is by thinking of it as a measure of how well the scanner preserves the average contrast of each input sine wave frequency. A modulation transfer of 1 indicates that the average contrast for a given sine wave frequency is perfectly maintained, while zero modulation transfer shows that it was completely lost, or not "seen" in the imaging process. Values between 0-1 indicate varying degrees of contrast preservation. Though there are exceptions, generally, the higher the modulation transfer, the better the preservation of detail by the imaging system.

Perhaps the best way to become grounded in interpreting MTF is through the use of example images captured using scanners with various MTFs. Figure 5 shows such an example for a section of an R.I.T. Alphanumeric Target.
The MTFs for each of the optical systems through which the target was imaged are labeled in the MTF plots below the image. Though the majority of the character sets are readable, their degree of legibility is quite different. The spatial image quality of the image on the right is superior to its two neighbors, and the middle one is superior to its left neighbor. This difference is reflected in the shape of the MTF. Figure 5 also illustrates why limiting resolution metrics derived from visual evaluations of classical targets of bars, characters, or other square wave-like features often fall short as spatial resolution tools.
Why is MTF better than other resolution measures?

Limiting resolution - If one were to use a limiting resolution technique, such as visual bar target readings on the examples of Fig. 5, each image would be rated the same since the smallest or limiting character group that can be just visually detected in each would be the "25E" group (near the bottom of the white square). These visual judgments of limiting resolution are considered threshold metrics because they rely on judgments of the threshold at which spatial resolution breaks down, and not the way it performs in getting to that threshold. Though useful in statistical analyses, one-bit imagery, and noise limited imaging, limiting resolution is not considered fundamental for resolution measurement. The three imaging systems illustrated in Fig.5 would have been judged equivalent with a limiting resolution metric. It is clear that they are not the same in terms of character image quality.

Dots Per Inch - For completeness, a few words on why specifying resolution of digital scanners in terms of dpi is misleading. The addressable resolution, specified in dots-per-inch (dpi), is marginally useful for evaluating true scanner resolution, but is only an indication of the sampling interval between pixels. It is not an indication of what size detail the scanner can ultimately see, or its MTF behavior, as much as it is of how finely the center of a pixel can be located.

For instance, it is reasonable to assume that a 400 dpi scanner will precisely sample a scanned document every 1/400". It is not reasonable to assume, however, that it will actually "see" details this small. It may, in fact, be able to detect details only as fine as a good 300 dpi scanner. In fact, for most desktop document scanners, the disparity between the sampling interval and actual detail detection becomes highly probable for claimed addressable resolutions beyond 1000 dpi.

It is like trying to compare the passing of either a needle (small cross section) or a large nail (large cross section) through common door screening. While both can be accurately located at any screen opening, because of its small size, only the needle will make it through. The same can be said of a digital document scanner. The sampling interval (screen spacing) does not determine how large or small an area that the scanner can see, but only how frequently it sees it.

Imaging System Analysis - Perhaps the single biggest benefit of using MTFs is for their utility in evaluating systems of connected imaging components. Besides the example of a scanner used in this paper, other system components, such as image processing, human vision, and information display, can also be cast in terms of MTF performance. Because MTF measurements are fundamental, the MTFs of these individual components can be cascaded to predict the way that a system of components will perform as a unit in terms of spatial resolution.

Conclusions
At a recent digitizing conference, one of the more prestigious speakers proclaimed, "Resolution is dead!" The message was that resolution was well understood with the real future challenges being consistent color reproduction. While the latter is no doubt a big hurdle, the trivializing of measuring spatial resolution as if it were a fashion trend is unfortunate, especially considering that most practitioners’ understanding of resolution stops at either limiting resolution or addressable resolution. The truly basic way of quantifying actual spatial resolution is through MTF analysis. It is one of the most important benchmarks for imaging performance in the scientific community because it is fundamental.
Software tools for easily evaluating the MTF of typical digital capture devices are on the verge of finding their way into the user community, and are currently being tested via efforts of standards committees. As of this writing these tools are not yet publicly available. When they are, their use will serve as a valuable and basic tool for true evaluation of spatial resolution performance. The reader can track the status of these standards and make inquiries through the I3A standards activities.